Detecting a physical difference between the CDM halos in simulation and in nature

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Numerical simulation is an important tool to help us understand the process of structure formation in the universe. However many simulation results of cold dark matter (CDM) halos on small scale are inconsistent with observations: the central density profile is too cuspy and there are too many substructures. Here we point out that these two problems may be connected with a hitherto unrecognized bias in the simulation halos. Although CDM halos in nature and in simulation are both virialized systems of collisionless CDM particles, gravitational encounter cannot be neglected in the simulation halos because they contain much less particles. We demonstrate this by two numerical experiments, showing that there is a difference on the microcosmic scale between the natural and simulation halos. The simulation halo is more akin to globular clusters where gravitational encounter is known to lead to such drastic phenomena as core collapse. And such artificial core collapse process appears to link the two problems together in the bottom-up scenario of structure formation in the ΛCDM universe. The discovery of this bias also has implications on the applicability of the Jeans Theorem in Galactic Dynamics.

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INTRODUCTION.

Thanks to rapid development in numerical techniques, the N-body simulation method has scored considerable success in furthering our understanding of the formation and evolution of the large-scale structures in the universe. But the simulations based on the CDM models has not worked so well on small scales, one aspect is the "cusp problem": The central density profile of the simulation halos (Fukushige & Makino 1997; Moore et al.1999; Ghigna et al. 2000; Navarro et al. 1996) are too cuspy, and are inconsistent with the observations on dwarf galaxies and low surface brightness galaxies (Gentile et al. 2004; Primack 2003). It is also inconsistent with the distribution of dark matter in galaxy clusters found from X-ray and gravitational lensing studies (Katayama et al. 2004; Tyson et al. 1998). Another aspect is that $10 \sim 100$ times more substructures are predicted than is observed (Willman et al. 2002; Hilker et al.2005);— the so-called "substructure problem". Some alternative models were introduced to deal with these problems, including the self-interaction dark matter(SIDM) model, the warm dark matter model etc. (Spergel & Steinhardt 2000; Götz et al. 2003; Boehm et al. 2002).

Here we realize that our understanding of these two problems can be greatly helped by studying a common, basic technical difficulty: the huge difference between the number of particles used in simulations of CDM halo, (N_{halo}) and the number occurring in nature: even in the most powerful "high-resolution" simulations today, a large dark matter halo contains only about 10^6-10^7 particles. Since we have to keep to a fixed mean density of the universe, the mass of each particle in the simulation has to be some 70 powers of ten the GeV candidates in particle physics (Bertone et al. 2001; Bertone et al. 2005)

It is hard to accept that the mass of the CDM particle can be as high as the mass of a small galaxy. We have unavoidably introduced a physical bias in our simulations which would impact on small scale problems. The bias is that we have simultaneously reduced the particle number density n and raised the mass of each particle m by a huge factor, $f_{AE} \sim 10^{70}$. CDM particle systems with such a physical bias can be expected to have very much biased dynamical and statistical properties, and the clustering and evolution scenario of CDM halos we learn from them might be different from reality. This problem should be discussed in more detail.

RELAXATION AND DETECTING METHOD

How will such bias affect the simulation results ? One effect is it changes the mean free path of two-body relaxation, L_s. For a virialized CDM halo with mass M and radius R, one can roughly estimate L_s as (Xiao et al. 2004):

$$L_s \simeq N_{halo} \frac{R}{3} \frac{\overline{\rho}}{\rho} \tag{1}$$

where $\overline{\rho} = M/(4\pi R^3/3)$ is the mean density of the halo, and ρ is its local density. For a virialized halo in nature that is composed of a huge number of GeV particles, it's easy to see from eq.(1) that $L_s \gg R$ and that we can completely disregard the two-body relaxation. However, for a virialized halo in simulation, composed of a much smaller number of huge mass particles, we have $L_s \sim R$, and the corresponding two-body scattering cross section is close to the value expected in the SIDM model. So, in the

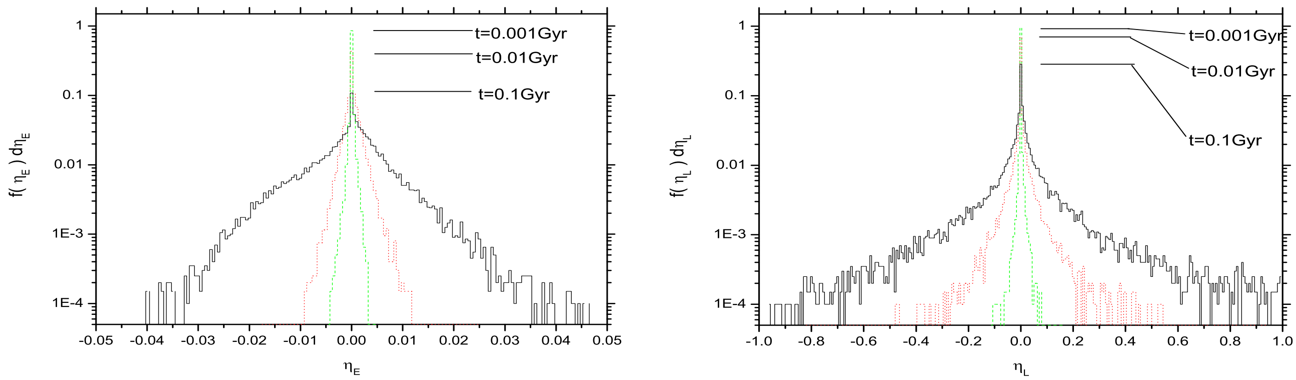

FIG. 1: Time evolution of the distribution function $f(\eta_{Ei}, t)$ and $f(\eta_{Li}, t)$ of a stable and spherical halo. Different from the expected delta function of a halo in nature, the $f(\eta, t)$ of the simulation halo in 0.001 Gyr(dash/green line), 0.01 Gyr(dot/red line) and 0.1 Gyr(solid/black line) is becoming more and more dispersive.

simulation, the effect of relaxation can no longer be neglected. Considering that star clusters that suffer similar relaxation can have their dynamical and statistical properties seriously affected, leading to core collapse, evaporation, etc. (Binney & Tremaine,1987), the discovery of this physical difference between CDM halos in simulation and in nature can help us understand the two small scale problems encountered in our current simulations.

In an effort to quantify and detect the relaxation process we have designed two numerical experiments for a spherical CDM halo. Different from the traditional statistical method (Binney & Knebe 2002; Diemand et al. 2004), we study the relaxation process from the viewpoint of the individual particle, and we find that the two integrals of motion, the energy and angular momentum of the ith particle of the halo, $E_i(\mathbf{x}, \mathbf{v})$ and $L_i(\mathbf{x}, \mathbf{v}) = |\mathbf{L}|$, can be effective detectors of the relaxation effect caused by the physical bias.

Although, a globular cluster and a stable spherical CDM halo in nature are both composed of collisionless gravitational interacting particles, their values of $L_s/R \sim N$ show that their gravitational potentials on small scale are very different. For a CDM halo in nature, gravitational encounter can be neglected and the halo potential $\phi(r)$ is stable and spherical for each CDM particle. Then E_i and L_i of each particle, being both integrals of motion, will be constant in such a system. In contrast, for a globular cluster or a CDM halo in simulation, $L_s \sim R$, and this means the collisionless particles in these systems have to experience gravitational encounters, and the potential ϕ is neither spherical nor stable. Then the E_i and L_i of each particle are no longer integrals of motion and are soon modified by encounters.—(The energy E and angular momentum L of the whole system are still the same. This is an important requirement for the reliability of many simulation results, but it is not relevant to the problem on hand.) From this point of view, the CDM halo in simulation is more like a globular cluster than a CDM halo in nature.

As measures of the relaxation effect in simulation CDM halos, we now define two parameters,

$$\eta_{Ei} = \frac{\Delta E_i}{|E_{i0}|} \ \ and \ \ \eta_{L_i} = \frac{\Delta L_i}{|L_{i0}|} \tag{2}$$

where $\Delta E_i = E_i(t) - E_{i0}$ and $\Delta L_i = L_i(t) - L_{i0}$ are the variations of E_i and L_i from their initial values E_{i0} and L_{i0}. For a CDM halo in nature, the distribution functions will simply be the time-independent delta-functions; for a stable, spherical halo in simulation, in contrast, they will be very different and will evolve in time.

SIMULATION AND RESULT OF NUMERICAL EXPERIMENTS

Here we present two controlled numerical experiments using the publicly available N-body tree code, GADGET (Springel et al., 2001), to detect the relaxation effect discussed above. Following Ma et al. (Gozdziewski et al.2005; Ma & Michael 2004; Michael & Ma 2004), we generate a stable spherical virialized CDM halo with a given NFW density profile (Navarro et al.1996):

$$\rho(r) = \rho_{crit}\overline{\delta}/[x(1+x)^2] \tag{3}$$

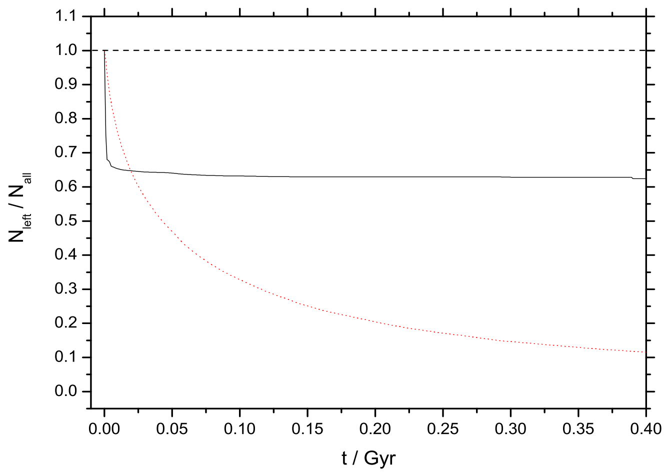

FIG. 2: Time evolution of N_{left}/N_{all} for E_i (solid/black line) and L_i (dot/red line). We can see the E_i and L_i have been changed for most of the particles in a short time, different from the expected dash line of the halo in nature that will not be affected by such relaxation.

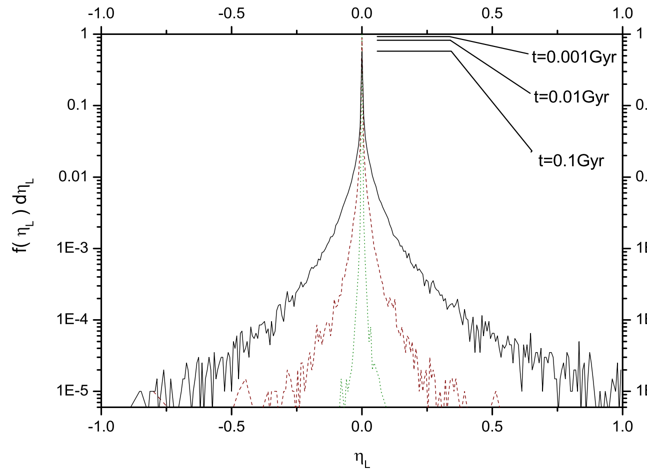

FIG. 3: Time evolution of the distribution function $f(\eta_{Li}, t)$ of a spherical evolving halo. Similar with the stable halo in Fig 1, the $f(\eta, t)$ of the simulation halo in 0.001 Gyr(dash/green line), 0.01 Gyr(dot/red line) and 0.1 Gyr(solid/black line) is becoming more and more dispersive.

where $x = r/r_s$,$\overline{\delta} = 200c^3/3[ln(1+c) - c/(1+c)]$, and $c = r_{vir}/r_s$ is the ratio of the halo's virial radius to scale radius. In all the runs, we set $r_s = 16kpc/h$, the total mass of the halo $M = 10^{12}M_{\odot}$, and we use a force softening of $\epsilon = 2kpc/h$. [1] We drew the particle velocities from a local isotropic Maxwellian distribution with the radius-dependent velocity dispersion computed from the Jeans equation.

In the first experiment, we start with a stable halo of 2×10^4 particles. We first let the individual particles of the NFW halo evolve for 5.0 Gyr, and we use the evolved halo as our initial condition, noting down the values E_{i0} and L_{i0} of each particle at this time. Then we record the evolution of E_i and L_i of each particle and obtain the distribution functions $f(\eta_{Ei}, t)$ and $f(\eta_{Li}, t)$ at chosen epochs, t. Theoretically, as integrals of motion, the E_i and L_i of each particle of such a halo in nature will be constant and the distribution function $f(\eta)$ should remain a delta function all the time. However, as we can see in Fig 1, both $f(\eta_{L_i})$ and $f(\eta_{E_i})$ are spreading out as time goes on. The spreading means that within a period of 0.1 Gyr, more and more particles have experienced changes in their E_i and L_i.

We define N_{left} as the number of particles which satisfy $|\eta_{Ei}| < 0.01$ or $|\eta_{Li}| < 0.01$ as measures of the evolution of the distribution functions. As we can see in Fig.2, for most particles in the simulated halo, E_i and L_i were changed very soon, and that the changes were faster in L_i than in E_i. In contrast, the value of N_{left}/N_{all} remains constant at 1.0 for a CDM halo in nature that consist of a huge number of GeV particles.

In the second experiment, we consider a spherically evolving halo. We multiply the velocity of each particle in the halo of the first experiment by a factor of 0.9 as the initial velocity. Then the halo will still be spherically symmetric, but it is no longer stable. In this case, E_i will be changed by the violent relaxation. Meanwhile, for a CDM halo in nature including numerous GeV collisionless particles, the L_i of each particle will remain as an integral of motion and will not change. Fig 3 shows the distribution function $f(\eta_{Li})$ of the simulation halo. Similar to Fig 1, the L_i of each particle changed rapidly due to this hitherto unsuspected process of galactic relaxation.

Finally, we tried to assess the effect of particle number on the distribution functions. We generated three systems with, respectively, 2×10^4, 2×10^5 and 2×10^6 particles, the other parameters remaining the same as for the stable halo of the first experiment. Fig 4 shows the time evolution of $\sigma^2_{\eta_L} \equiv \langle \eta^2_{Li} \rangle$. We find that in all cases $\sigma^2_{\eta_L}$ increases as a power-law function of time (within about $0.1Gyr$), and, at any given time, $\sigma^2_{\eta_L}$ is a fast decreasing function of N. In other words, the relaxation effect, or the bias, is greatly reduced when N is increased.

[1] Our test runs showed that different values of $\rho(r)$ and ϵ etc. do have a small effect on the results, but will not change the general qualitative picture.

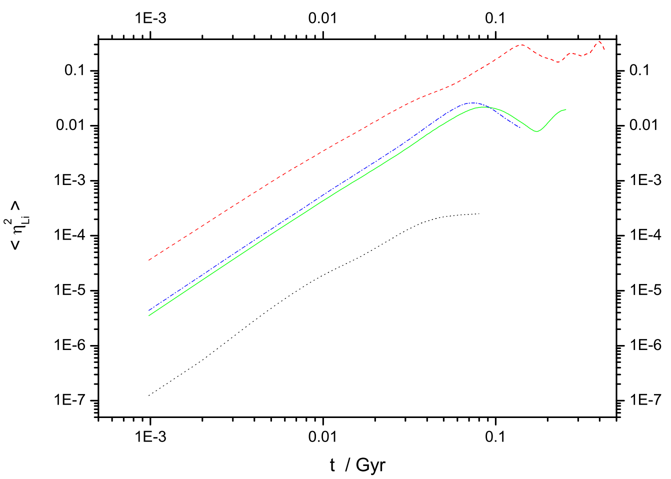

FIG. 4: $\sigma^2_{\eta_L} \equiv \langle \eta^2_{Li} \rangle$ as a function of time, for stable spherical halos with different particle numbers: 2×10^4 (dash/red line), 2×10^5(solid/green line), 2×10^6(dot/black line) and the spherical evolving halo with 2×10^5(dash dot/blue line) particles.

DISCUSSION AND CONCLUSION

What is the difference?— Our demonstration with the two numerical experiments brings out a physical difference in microcosm between dark matter halos in simulations and in nature: although they are both composed of collisionless CDM particles, the process of gravitational encounter can be neglected in natural halos, but not in simulation halos.

How will it affect the simulation results?— On small scale, the relaxation can greatly affect the behavior of individual particles in simulation halos. Take the case of the stable halo as an example. For the halo as a whole, the total energy E and the total angular momentum L are each conserved. What's more, statistically $\phi(r)$ is still a static and spherically symmetric potential in a short period. However Fig 1 and Fig 2 clearly show changes of E_i and L_i caused by gravitational encounters in a microstatistical sense. In other words, from the point of view of the individual particle of the simulation halo, the potential is *nolonger* spherically symmetric or static. To be exact, for a halo that comprises fewer than approximately 10^6 particles, gravitational collision must be taken into count. The halo is no longer a system of collisionless particles, and the collisionless Boltzmann Equation is no longer applicable.

Will the general dynamical behavior of the whole system in the large scale statistics respect, be affected by such microcosmic effect? The answer is obviously yes. For the CDM halo in simulation, the E_i and L_i of each particle can vary, which means the distribution function $f(E_i, L_i)$ of the whole system can be different from the natural case. And if we can't trust the $f(E_i, L_i)$ in simulation, then the density profile $\rho(r)$ and velocity dispersion of the halo we learn from the simulation will be subject to doubt. Actually, whether we can express the distribution function $f(\mathbf{x}, \mathbf{v})$ in the form of $f(E, L)$ needs to be considered (see the following discussion about the Jeans Theorem). There isn't any mature theory dealing with the effect of such microcosmic relaxation on $\rho(r)$ so far, and more research is expected.

However, by comparing our results above with a previous research on dynamic behavior of stellar clusters (Binney & Tremaine 1987), we can learn a lot about how the difference caused by the particle number will affect the simulation results. Much research has been done on stellar systems (such as globular clusters) which comprise similar numbers of collisionless particles under pure gravitational interaction. The dynamical effects caused by the process of gravitational encounter in stellar systems can also appear in simulation halos. A good example is the core collapse process: due to star-star encounters in the core, some of the stars get very high energies and evaporate away, while the core shrinks as required by energy conservation. Following the way of studying stellar systems, we can also estimate the time to core collapse τ of one CDM halo (Binney & Tremaine 1987, eq. 8-72) as:

$$\tau \simeq 20 t_{rh} \simeq \frac{1.3 \times 10^{10} yr}{ln(0.4N)} (\frac{M}{10^{12} M_\odot})^{1/2} (\frac{1 M_\odot}{10^8 m}) (\frac{r_h}{10kpc})^{3/2} \quad (4)$$

Where M and r_h are the total mass and median radius of the halo, m is the mass of each particle of it, and t_{rh} is the median relaxation time of the system . Now we can see, the core collapse time scale of an isolated spherical simulation CDM halo with $M \simeq 10^{12} M_\odot$,$r_h \simeq 10kpc$ and including $N \simeq 10^4$ particles ($m = M/N \simeq 10^8 M_\odot$), will be about the age of the universe. Based on the estimation above, one might expect the CDM halo in simulation can avoid such core collapse process. Unfortunately, considering:

(1) According to the hierarchical structure formation scenario in a ΛCDM universe, small structures come into being first and large halos comes from the merging of small ones.

(2) In the same simulation, with the same value of m above but much smaller M and r_h, eq.(4) indicates small halos will have a much shorter core collapse time scale than the age of the universe(one can also see from Fig.4 and eq.(1)). So these small halos in simulation will have to suffer the core collapse process, and form one artificial cusp center. On the contrary, such core collapse will not happen in a CDM halo in nature which includes very large number of small $m \sim GeV$ particles, for they have

a time scale τ much longer than 10^{10} years.

(3)Numerical studies (Hayashi et al.2003, Michael & Ma 2004) have shown the cuspy centers can survive from merging process of both the major mergers of galaxy halos and the evolution of substructures.

Now we can figure out how do the bias affect the simulation results: In a ΛCDM universe, small halos come into being first and the unexpected core collapse process lead to the artificial cuspy centers of them. Then these cuspy centers survive from the merging process later and contribute to both the "cusp problem" and the "substructure problem". No matter if there is other explanation for these problems, the contribution of this effects must be taken into account when we analyze simulation results on small scales. One can even predict, with the improvement of simulation technology, the "substructure problem" can be even more serious if the particle number increased by only one or two orders.

Last but not least, our discovery of the bias directly tests the premise of the commonly used Jeans Theorem in galactic dynamics. According to the Jeans Theorem, in a static and spherical symmetric potential well, we can express the distribution function $f(\mathbf{x}, \mathbf{v})$ in phase space as $f(E, L)$ only when the E_i and L_i of each particle are all integrals of motion. Our result shows that, for the collisionless particle system with pure gravitational interaction which has a particle number $N \leqslant 10^6$, gravitational encountering can not be dismissed. As a result, the E_i and L_i for most of the particles will be changed in a short period of time (see Fig. 2). For such systems like CDM halos in simulation and globular clusters, the E_i and L_i are no longer integrals of motion and the Jeans Theorem is no longer applicable, while it is still valid for CDM halos in nature that are made up of a huge number of GeV particles.

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